

OCHA-PP-197
hep-ph/0212242

Neutrino Mass Matrix Out of Up-Quark Masses Only

Masako BANDO* and Midori OBARA†

* *Aichi University, Aichi 470-0296, Japan*† *Graduate School of Humanities and Sciences,
Ochanomizu University, Tokyo 112-8610, Japan*

Abstract

Under the assumption of symmetric four zero texture for fermion mass matrices in $SO(10)$ model, the neutrino Dirac mass matrix is derived from the up-quark mass matrix. By adjusting two scales of the Majorana mass sector, we can derive observed neutrino two large mixing angles very naturally. It is indeed remarkable that all the masses and mixings of neutrinos are expressed in terms of only three known parameters, m_t, m_c, m_u . A typical example of our model is

$$\tan^2 2\theta_{\tau\mu} = \frac{4}{(1 - 2\frac{m_c}{\sqrt{m_u m_t}})^2}.$$

*E-mail address: bando@aichi-u.ac.jp

†E-mail address: midori@hep.phys.ocha.ac.jp

Recent neutrino experiments by Super-Kamiokande [1, 2] and SNO [3] have confirmed neutrino oscillations with large mixing angles:

$$\begin{aligned}\sin^2 2\theta_{atm} &\leq 0.83 (99\% C.L.), \\ \tan^2 \theta_{sol} &= 0.24 - 0.89 (99.73\% C.L.),\end{aligned}\tag{1}$$

with mass squared differences of

$$\Delta m_{atm}^2 \sim 2.5 \times 10^{-3} \text{ eV}^2,\tag{2}$$

$$\Delta m_{sol}^2 \sim 5 \times 10^{-5} \text{ eV}^2.\tag{3}$$

The neutrino mixing angles are expressed by MNS matrix [4] which is written as

$$V_{MNS} = U_l^\dagger U_\nu,\tag{4}$$

with U_l and U_ν being the unitary matrices which diagonalize the 3×3 charged lepton and neutrino mass matrices, M_l and M_ν ,

$$U_l^\dagger M_l^\dagger M_l U_l = \text{diag}(m_e^2, m_\mu^2, m_\tau^2),\tag{5}$$

$$U_\nu^\dagger M_\nu^\dagger M_\nu U_\nu = \text{diag}(m_{\nu_e}^2, m_{\nu_\mu}^2, m_{\nu_\tau}^2),\tag{6}$$

respectively. Here the left-handed neutrino mass matrix is expressed in terms of right-handed Majorana neutrino mass matrix, M_R , and Dirac neutrino mass matrix, $M_{\nu D}$,

$$M_\nu = M_{\nu D}^T M_R^{-1} M_{\nu D}.\tag{7}$$

If we restrict ourselves to the case in which such large mixings are naturally derived without fine tuning, the origin of each of the large mixing angles, $\theta_{\mu\tau}$ and $\theta_{e\mu}$, must be due to either M_ν or M_l .

In this paper, we show that these two large mixing angles can be derived from symmetric four zero texture within the $SO(10)$ GUT framework. We assume the following textures for up- and down-type quark mass matrices at the GUT

scale [5],[‡]

$$\begin{aligned}
M_D &= \begin{pmatrix} 0 & \sqrt{\frac{m_d m_s m_b}{m_b - m_d}} & 0 \\ \sqrt{\frac{m_d m_s m_b}{m_b - m_d}} & m_s & \sqrt{\frac{m_d m_b (m_b - m_s - m_d)}{m_b - m_d}} \\ 0 & \sqrt{\frac{m_d m_b (m_b - m_s - m_d)}{m_b - m_d}} & m_b - m_d \end{pmatrix} \\
&\simeq m_b \begin{pmatrix} 0 & \frac{\sqrt{m_d m_s}}{m_b} & 0 \\ \frac{\sqrt{m_d m_s}}{m_b} & \frac{m_s}{m_b} & \sqrt{\frac{m_d}{m_b}} \\ 0 & \sqrt{\frac{m_d}{m_b}} & 1 \end{pmatrix}
\end{aligned} \tag{8}$$

$$M_U \simeq m_t \begin{pmatrix} 0 & \frac{\sqrt{m_u m_c}}{m_t} & 0 \\ \frac{\sqrt{m_u m_c}}{m_t} & \frac{m_c}{m_t} & \sqrt{\frac{m_u}{m_t}} \\ 0 & \sqrt{\frac{m_u}{m_t}} & 1 \end{pmatrix} \equiv m_t \begin{pmatrix} 0 & a & 0 \\ a & b & c \\ 0 & c & 1 \end{pmatrix}. \tag{9}$$

As for M_D it is well known that each elements of M_U and M_D is dominated by the contirbution either from **10** or **126** Higgs fields, where the ratio of Yukawa couplings of charged lepton to down quark are 1 or -3 , respectively. More concretely the following option for M_D (Georgi-Jarlskog type [6])

$$M_D = \begin{pmatrix} 0 & \mathbf{10} & 0 \\ \mathbf{10} & \mathbf{126} & \mathbf{10} \\ 0 & \mathbf{10} & \mathbf{10} \end{pmatrix}, \tag{10}$$

is known to reproduce very beautifully all the experimental data of m_τ, m_μ, m_e as well as m_b, m_s, m_d [7, 8]. On the other hand, M_U is related to M_{ν_D} , which is not directly connected to neutrino experiments no definite configuration has been found so far. Here we show the following option reproduces two large mixings at the same time,

$$M_U = \begin{pmatrix} 0 & \mathbf{126} & 0 \\ \mathbf{126} & \mathbf{10} & \mathbf{10} \\ 0 & \mathbf{10} & \mathbf{126} \end{pmatrix}, \tag{11}$$

[‡]Here we neglect the CP phases, since they have little effect on the final result. The simple expressions Eq. (8) and (9) are derived by using $m_u \ll m_c \ll m_t$, $m_d \ll m_s \ll m_b$.

which uniquely determines neutrino Dirac mass matrix as

$$M_{\nu_D} = m_t \begin{pmatrix} 0 & -3a & 0 \\ -3a & b & c \\ 0 & c & -3 \end{pmatrix}. \quad (12)$$

For the right-handed Majorana mass matrix, to which only **126** Higgs field couples, we assume the following option consistently with M_U :

$$M_R \equiv m_R \begin{pmatrix} 0 & r & 0 \\ r & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}. \quad (13)$$

Then from Eq. (7), we get

$$M_\nu = \begin{pmatrix} 0 & \frac{a^2}{r} & 0 \\ \frac{a^2}{r} & \frac{-2ab}{3r} + \frac{c^2}{9} & -\frac{c}{3}(\frac{a}{r} + 1) \\ 0 & -\frac{c}{3}(\frac{a}{r} + 1) & 1 \end{pmatrix} \frac{9m_t^2}{m_R}. \quad (14)$$

Since the order of the parameters in the above Eq. (14) are $a \ll b \leq c \ll 1$, we recognize that the first term of the 3-2 element, $\frac{-3ac}{r}$, must be of order 1 in order to get large mixing angle $\theta_{\mu\tau}$, so we here take

$$r \simeq \frac{ac}{3} \simeq \frac{\sqrt{m_u^2 m_c}}{3\sqrt{m_t^3}} \sim 10^{-7}. \quad (15)$$

This is almost the same situation as discussed by Kugo, Yoshioka and one of the present authors [9]. This tiny value of r is very welcome [9]; the right-handed Majorana mass of the third generation must become of the order of GUT scale while those of the first and second generations are of order 10^8 GeV. This is quite favorable for the GUT scenario to reproduce the bottom-tau mass ratio.

Up to here the situation is quite trivial in a sense; one arbitrary parameter r has been chosen so as to reproduce the large mixing angle $\theta_{\mu\tau}$. Now the problem is whether it naturally reproduces another mixing angle $\theta_{e\mu}$. At this stage we have no arbitrary parameter to adjust the mass ratios or mixing angles. Under such

condition, M_ν is approximately written as

$$M_\nu = \begin{pmatrix} 0 & \frac{-3a}{c} & 0 \\ \frac{-3a}{c} & \frac{2b}{c} & -1 \\ 0 & -1 & 1 \end{pmatrix} \frac{9m_t^2}{m_R} \equiv \begin{pmatrix} 0 & \beta & 0 \\ \beta & \alpha & -1 \\ 0 & -1 & 1 \end{pmatrix} \frac{9m_t^2}{m_R}, \quad (16)$$

with $\alpha = \frac{2b}{c} = \frac{2m_c}{\sqrt{m_t m_u}}$ and $\beta = \frac{-3a}{c} = -3\sqrt{\frac{m_c}{m_t}}$. Since $\beta \ll \alpha \sim 1$, we first diagonalize the 2×2 matrix of the 2-3 block of Eq. (16) by rotating the following angle

$$\tan^2 2\theta_{\mu\tau} = \frac{4}{(1 - \alpha)^2}, \quad (17)$$

through which M_ν is deformed as follows

$$\begin{pmatrix} 0 & \beta & 0 \\ \beta & \alpha & -1 \\ 0 & -1 & 1 \end{pmatrix} \rightarrow \begin{pmatrix} 0 & \beta \cos \theta_{\mu\tau} & \beta \sin \theta_{\mu\tau} \\ \beta \cos \theta_{\mu\tau} & \lambda_2 & 0 \\ \beta \sin \theta_{\mu\tau} & 0 & \lambda_3 \end{pmatrix}, \quad (18)$$

with

$$\lambda_3 = \frac{\alpha + 1 + \sqrt{(\alpha - 1)^2 + 4}}{2} (\simeq \lambda_\tau), \quad (19)$$

$$\lambda_2 = \frac{\alpha + 1 - \sqrt{(\alpha - 1)^2 + 4}}{2}. \quad (20)$$

Then we further diagonalize the 2×2 matrix of the 1-2 block of Eq. (18) by rotating $\theta_{e\mu}$:

$$M_\nu \rightarrow \begin{pmatrix} \lambda_e & 0 & \beta \sin \theta_{\mu\tau} \cos \theta_{e\mu} \\ 0 & \lambda_\mu & \beta \sin \theta_{\mu\tau} \sin \theta_{e\mu} \\ \beta \sin \theta_{\mu\tau} \cos \theta_{e\mu} & \beta \sin \theta_{\mu\tau} \sin \theta_{e\mu} & \lambda_\tau \end{pmatrix}, \quad (21)$$

with the rotating angle

$$\tan^2 2\theta_{e\mu} = \left(\frac{2\beta \cos \theta_{\mu\tau}}{\lambda_2} \right)^2, \quad (22)$$

and mass eigenvalues

$$\lambda_\mu = \frac{\lambda_2 + \sqrt{\lambda_2^2 + 4\beta^2 \cos^2 \theta_{\mu\tau}}}{2}, \quad (23)$$

$$\lambda_e = \frac{\lambda_2 - \sqrt{\lambda_2^2 + 4\beta^2 \cos^2 \theta_{\mu\tau}}}{2}. \quad (24)$$

Finally we get the following approximate form for the rest small rotating angle,

$$\sin \theta_{e3} = \frac{2\beta \sin \theta_{\mu\tau} \cos \theta_{e\mu}}{\lambda_\tau}. \quad (25)$$

Leaving details in a separate paper [10], we demonstrate how we can predict neutrino masses and mixings; all the neutrino information are determined in terms of m_u, m_c, m_t as

$$\tan^2 2\theta_{\mu\tau} \simeq \frac{1}{\left(1 - \frac{2m_c}{\sqrt{m_u m_t}}\right)^2}, \quad (26)$$

$$\tan^2 2\theta_{e\mu} \simeq \frac{9m_c}{m_t \left(1 - \frac{2m_c}{\sqrt{m_u m_t}}\right)^2}, \quad (27)$$

$$\sin \theta_{e3} \simeq -3\sqrt{\frac{m_c}{m_t}} \sin \theta_{\mu\tau} \cos \theta_{e\mu}, \quad (28)$$

from which the following equations are derived

$$\tan^2 2\theta_{e\mu} \simeq \frac{9m_c}{m_t} \tan^2 2\theta_{\mu\tau}, \quad \sin^2 \theta_{e3} \simeq \frac{9m_c}{m_t} \sin^2 \theta_{\mu\tau} \cos^2 \theta_{e\mu}. \quad (29)$$

This indicates that $\tan^2 2\theta_{e\mu}$ is smaller by a factor $\frac{9m_c}{m_t}$ than $\tan^2 2\theta_{\mu\tau}$. Interesting enough is that once we know the atmospheric and solar neutrino experiments, U_{e3} is predicted without any ambiguity coming from the up-quark masses at GUT scale;

$$\sin^2 \theta_{e3} \simeq \frac{\tan^2 2\theta_{e\mu}}{\tan^2 2\theta_{\mu\tau}} \cdot \sin^2 \theta_{\mu\tau} \cos^2 \theta_{e\mu}, \quad (30)$$

which is independent of the uncertainty especially coming from the value, m_t , at GUT scale.

Next the neutrino masses are given by

$$m_{\nu_\tau} \simeq \lambda_\tau \cdot \frac{m_t^2}{m_R}, \quad m_{\nu_\mu} \simeq \lambda_\mu \cdot \frac{m_t^2}{m_R}, \quad m_{\nu_e} \simeq \lambda_e \cdot \frac{m_t^2}{m_R}, \quad (31)$$

where the renormalization factor ($\sim \frac{1}{3}$) has been taken account to estimate the lepton masses at low energy scale. Since $\lambda_\mu \ll \lambda_\tau \sim O(1)$, this indeed yields $m_R \sim 10^{16}$ GeV, as many people require. On the other hand, m_{ν_μ} is expected to become almost of the same order to m_{ν_e} as is seen from Eq. (31).

Let us make some numerical calculation. We take the values of m_t, m_c, m_u at GUT scale obtained by Fusaoka and Koide [11];

$$m_u = 1.04_{-0.20}^{+0.19} \text{ MeV}, \quad (32)$$

$$m_c = 302_{-27}^{+25} \text{ MeV}, \quad (33)$$

$$m_t = 129_{-40}^{+196} \text{ GeV}. \quad (34)$$

Among them m_t is the most sensitive parameter. Fig. 1 and Fig. 2 show the dependence of the resultant values of $\theta_{\mu\tau}$ and $\theta_{e\mu}$ on m_t , respectively. From Fig. 1 m_t is found to be larger than 90 GeV and from Fig. 2 the lower bound is $m_t = 170$ GeV. With this bound for m_t (170 – 320 GeV) we can predict the values of U_{e3} ,

$$U_{e3} \sim 0.03 - 0.045. \quad (35)$$

from Fig. 3. We hope this can be checked by experiment in near future JHF-Kamioka long-base line [12], the sensitivity of which is reported as $|U_{e3}| \simeq 0.04$ at 90 % C.L.. If we further expect Hyper-Kamiokande ($|U_{e3}| < 10^{-2}$) [13], we can completely check whether such symmetric texture model can survive or not. In conclusion we list a set of typical values of neutrino masses and mixings at $m_t \simeq 240$ GeV;

$$\sin^2 2\theta_{\mu\tau} \sim 0.95 - 1, \quad (36)$$

$$\tan^2 \theta_{e\mu} \sim 0.23 - 0.6, \quad (37)$$

$$U_{e3} \sim 0.037 - 0.038, \quad (38)$$

$$m_{\nu_\tau} \sim 0.06 - 0.07 \text{ eV}, \quad (39)$$

$$m_{\nu_\mu} \sim 0.003 - 0.006 \text{ eV}, \quad (40)$$

$$m_{\nu_e} \sim 0.0007 - 0.0015 \text{ eV}, \quad (41)$$

with $m_R = 2 \times 10^{15}$ GeV and $rm_R = 10^8$ GeV, which correspond to the Majorana mass for the third generation and those of the second and first generations, respectively.

Remark that, once the scale of right-handed Majorana mass matrix, r , is determined so as for the mixing angle of atmospheric neutrino to become maximal, the same value r well reproduces the ratio of the mass differences $\Delta m_{\mu\tau}^2$ to $\Delta m_{e\mu}^2$.

We add two comments. First, one might suspect that we may always reproduce any desired neutrino mass matrix by adjusting the parameters appearing in M_R , namely we can take $M_R = M_{\nu_D} M_\nu^{-1} M_{\nu_D}^T$. However this is not actually true if the mass matrix M_U is of such hierarchical structure as those coming from the famous

anomalous $U(1)$. We shall show this in a separate paper [10], where other options of Eq. (11) are fully investigated to confirm that only the option adopted in this paper can reproduce the two large mixing angles. Second, our scenario is quite different from those starting from the assumption that the large mixing angles observed in neutrino oscillation data comes from the charged lepton mass matrix; one might predict some relations of neutrino mixing angles to down quark information [14], but we would no more predict the absolute values of neutrino masses, which indeed needs the information of M_ν . Our scenario, if it is indeed true, can predict without any ambiguity even for the order-one coefficients. The remarkable results are obtained really thanks to the power of GUT.

Acknowledgements

This work started from the discussion at the research meeting held in Nov. 2002 supported by the Grant-in Aid for Scientific Research No. 09640375. We would like to thank to A. Sugamoto and T. Kugo whose stimulating discussion encouraged us very much. Also we are stimulated by the fruitful and instructive discussions during the Summer Institute 2002 held at Fuji-Yoshida. M. B. is supported in part by the Grant-in-Aid for Scientific Research Nos. 12640295 from Japan Society for the Promotion of Science, and Grants-in-Aid for Scientific Research on Priority Area A “Neutrinos” (Y. Suzuki) Nos. 12047225, from the Ministry of Education, Science, Sports and Culture, Japan. Also we are thankful to T. Takeuchi for his kind comment on our poor English.

References

- [1] Super-Kamiokande Collaboration, S. Fukuda *et al.*, Phys. Lett. **B433** (1998) 9; *ibid.* **436**, 33 (1998); *ibid.* **539**, 179 (2002).
- [2] Super-Kamiokande Collaboration, S. Fukuda *et al.*, Phys. Rev. Lett. **86** (2001) 5651; *ibid.* **86**, 5656 (2001).

- [3] SNO Collaboration, Q.R. Ahmad *et al.*, Phys. Rev. Lett. **89** (2002) 011301; *ibid.* **89**, 011302 (2002).
- [4] Z. Maki, M. Nakagawa and S. Sakata, Prog. Theor. Phys. **28** (1962) 870.
- [5] H. Nishiura, K. Matsuda and T. Fukuyama, Phys. Rev. **D60** (1999) 013006.
- [6] H. Georgi and C. Jarlskog, Phys. Lett. **B86** (1979) 297.
- [7] J. Harvey, P. Ramond and D. Reiss, Phys. Lett. **B92** (1980) 309; Nucl. Phys. **B199** (1982) 223.
- [8] S. Dimopoulos, L.J. Hall and S. Raby, Phys. Rev. Lett. **68** (1992) 1984; Phys. Rev. **D45** (1992) 4195.
- [9] M. Bando, T. Kugo and K. Yoshioka, Phys. Rev. Lett. **80** (1998) 3004.
- [10] to appear
- [11] H. Fusaoka and Y. Koide, Phys. Rev. **D57** (1998) 3986.
- [12] Y. Itow *et al.*, arXiv: hep-ex/0106019.
- [13] M. Aoki, K. Hagiwara, Y. Hayato, T. Kobayashi, T. Nakaya, K. Nishikawa and N. Okamura, arXiv: hep-ph/0112338.
- [14] M. Bando, T. Kugo and K. Yoshioka, Phys. Lett. **B483** (2000) 163.

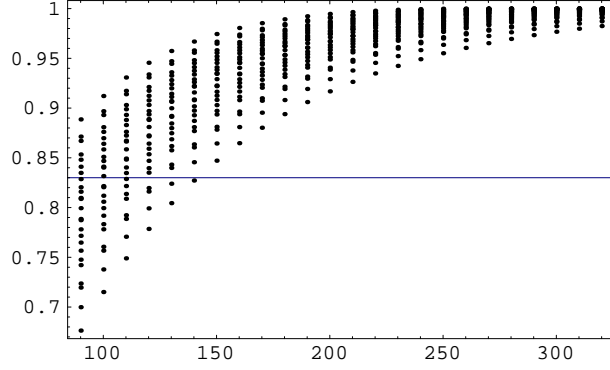


Figure 1: Calculated values of $\sin^2 2\theta_{\mu\tau}$ versus m_t . The parameter region of m_t at GUT scale is within the allowed uncertainty. The horizontal line indicates the experimental lower bound for atmospheric neutrinos.

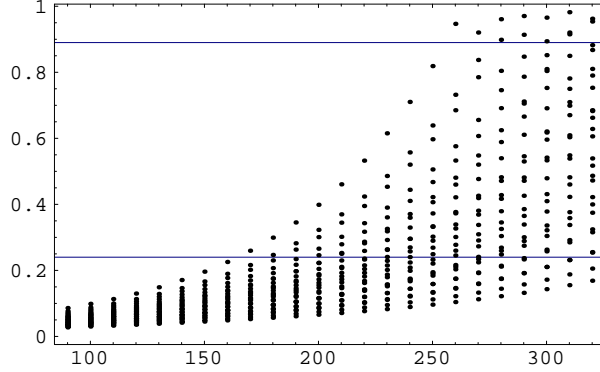


Figure 2: $\tan^2 \theta_{e\mu}$ as function of the top quark mass, m_t . The parameter region of m_t is taken within the uncertainty of m_t at GUT scale. The horizontal lines indicate the experimental allowed regions for solar neutrinos.

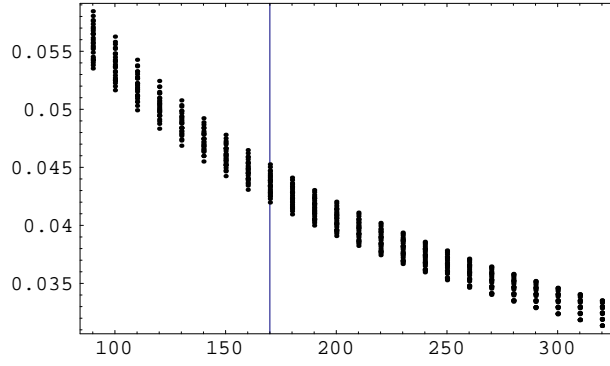


Figure 3: Predicted values of U_{e3} versus m_t at GUT scale within uncertainty. The vertical line indicates the point $m_t = 170$ GeV.